RADNNET-MBL: A NEURAL NETWORK APPROACH FOR EVALUATION OF ABSORPTIVITY AND EMISSIVITY OF NON-GRAY COMBUSTION GAS MIXTURE BETWEEN FINITE AREAS AND VOLUMES

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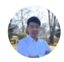

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RADNNET-MBL: A NEURAL NETWORK APPROACH FOR EVALUATION OF ABSORPTIVITY AND EMISSIVITY OF NON-GRAY COMBUSTION GAS MIXTURE BETWEEN FINITE AREAS AND VOLUMES

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ABSTRACT

RADNNET-MBL (RADiation-Neural-NETwork with Mean-Beam-Length) is developed to provide a computationally efficient and accurate method for the evaluation of radiative heat transfer between arbitrary rectangular surfaces in a three-dimensional enclosure with a non-gray combustion medium. Exact mean beam lengths between an infinitesimal area and a finite rectangular area are calculated for a wide range of geometric configurations and optical thicknesses. For a specific geometric configuration, the effect of optical thickness on mean beam lengths is examined. Based on numerical experiment, a constant averaged mean beam length is defined and shown to be effective in generating accurate prediction of the transmissivity over all optical thicknesses. Utilizing the averaged mean beam lengths together with RADNNET (a neural network-based correlation), exchange factors between two finite rectangular areas with an intervening non-gray combustion mixture can readily be obtained. A case study is presented. Results show that RADNNET-MBL provides promising accuracy (with absolute error less than 1%) with a significant reduction in computational effort.

KEYWORDS: Mean beam length, neural network, non-gray, three-dimensional, radiation heat transfer

1. INTRODUCTION

The evaluation of radiative heat transfer with the presence of a participating medium consisted of combustion products (i.e., H_2O, CO_2, and soot particulate) in a three-dimensional enclosure is numerically complex. Even for a one-dimensional isothermal homogeneous medium, the absorptivity is a complicated function of six independent variables, including source temperature, mixture temperature, soot volume fraction, and optical thickness of H_2O, CO_2, and CO [1]. In order to account for the spectral behavior of the gaseous mixture, a direct numerical integration using realistic spectral data is required. To account for the geometric effect between arbitrary surfaces and the medium, another direct numerical integration is also needed. Previous work [2] shows that for a three dimensional rectangular enclosure, the determination of necessary exchange factors will require 480 million numerical evaluations for each time-step in a typical engineering calculation. In the area of fire protection engineering, a common practice to overcome this numerical bottleneck is to utilize simplified methods [3,4]. These simplified methods have never been benchmarked with exact solutions and the accuracy of these methods is therefore highly uncertain. Recent studies [5,6] have shown that the use of such simplified methods can potentially lead to substantial errors (higher than 100 % or up to 1000 %). Indeed, the lack of a mathematically validated and computational efficient methodology which would allow non-radiation experts to implement the correct physics of radiative transfer into practical engineering design calculations is a serious obstacle, particularly to the fire protection community, in understanding the effect of radiative heat transfer for fire safety consideration.

In previous works, using realistic spectral data of RADCAL [7] (a narrow band model), a neural network based model, RADNNET, was developed [1] and was shown to be a computational efficient approach to determine the total absorption characteristics of a one-dimensional non-gray combustion gas mixture. RADNNET was subsequently extended to RADNNET-ZM [8], which evaluates total emissivity and absorptivity of a

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homogeneous combustion medium in a three-dimensional rectangular enclosure by direct numerical integrations. The objective of this paper is to show that by using the concept of mean beam length (MBL), the computational efficiency for the evaluation of the total emissivity and absorptivity of a three-dimensional non-gray combustion gas mixture can be significantly improved. In the following sections, the mathematical formulation of MBL, for two fundamental rectangular geometries is first presented. While MBL is generally a function of optical thickness, numerical data will be generated to show that an average MBL can be defined and used to correlate the absorption effect over all optical thicknesses to a high degree of accuracy. Using superposition and the tabulated average MBL for the two fundamental rectangular geometric configurations, RADNNET-MBL is developed. Numerical results are generated to demonstrate both the accuracy and numerical efficiency of RADNNET-MBL.

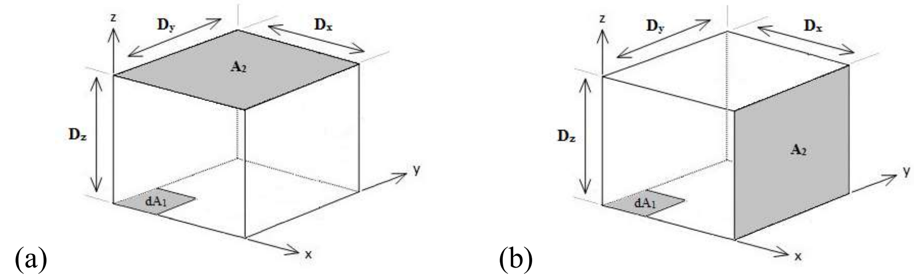

Fig 1. Exchange factor configuration for a) parallel areas and b) perpendicular areas.

2. MATHEMATICAL FORMULATION

Consider a one-zone enclosure filled with a homogenous mixture of water vapor, carbon dioxide, and soot particulate with arbitrary dimensions of D_x, D_y, and D_z as shown in Figs. 1a and 1b, the analysis of radiative heat transfer to the bounding surfaces requires the evaluation of surface-surface exchange factors. Specifically, for perpendicular areas as shown in Fig. 1b (the mathematical development for the geometry shown in Fig. 1a is similar and will not be presented in this paper due to page limitation, the derivation will be available to the reader upon request), the radiative heat transfer from a differential area dA_1 to area A_2, Q_{d1-2}, is given by

$$Q_{d1-2} = \frac{W_{d1} dA_1}{\pi} \iint \frac{e^{-kr} D_x D_z}{r^4} dA_2 = W_1 ds_1 s_2 \tag{1}$$

where W_1 is the radiosity leaving the differential area dA_1, r is the center-to-center distance between the differential area and the finte area A_2, and k is the loacl absorption coefficient of the mixture. From Eq. (1), the exchange factor can be rewitten as

$$ds_1 s_2 = \frac{dA_1}{\pi} \iint \frac{e^{-kr} D_x D_z}{r^4} dA_2 = dA_1 F_{d1-2} \tau_{d1-2} \tag{2}$$

with

$$F_{d1-2} \tau_{d1-2} = \iint \frac{e^{-kr} D_x D_z}{\pi r^4} dA_2 \tag{3}$$

and $r = \left(D_z^2 + D_y^2 + D_x^2\right)^{\frac{1}{2}}$ and $dA_2 = dzdy$. Let $s = \left(D_z^2 + D_y^2\right)^{\frac{1}{2}}$ and $dA_2 = sdsd\phi$ with $D_z = scos\phi$, Eq. (3) can be written in polor coordinate as

$$F_{d1-2} \tau_{d1-2} = \int_0^{S_{max}} \int_{\phi_{min}}^{\phi_{max}} \frac{e^{-kr} D_x scos\phi}{\pi r^4} sdsd\phi \tag{4}$$

Eq. (4) can be further integrated in the angular direction for different range of variables, leading to three one-dimensiional integrals as follow

$$F_{d1-2} \tau_{d1-2} = g_1 + g_2 + g_3 \tag{5}$$

with
$$g_1 = \int_0^{\frac{D_y}{D_x}} \frac{e^{-kD_x\sqrt{1+\eta^2}}}{\pi(1+\eta^2)^2}\eta^2 d\eta \tag{6a}$$

$$g_2 = \frac{D_y}{\pi D_x}\int_{D_y/D_x}^{\sqrt{\frac{D_y^2}{D_x^2}+\frac{D_z^2}{D_x^2}}} \frac{e^{-kD_x\sqrt{1+\eta^2}}}{(1+\eta^2)^2}\eta d\eta \tag{6b}$$

$$g_3 = -\frac{1}{\pi}\int_{\frac{D_z}{D_x}}^{\sqrt{\frac{D_y^2}{D_x^2}+\frac{D_z^2}{D_x^2}}} \frac{e^{-kD_x\sqrt{1+\eta^2}}}{(1+\eta^2)^2}\sqrt{\eta^2-\frac{D_z^2}{D_x^2}}\,\eta d\eta \tag{6c}$$

and $\eta = s/D_x$. Note that F_{d1-2} is the view factor between dA_1 and A_2 given by

$$F_{d1-2} = \frac{1}{2\pi}\left[tan^{-1}\left(\frac{D_y}{D_x}\right) - \frac{1}{\sqrt{1+\frac{D_z^2}{D_x^2}}}tan^{-1}\left(\frac{D_y}{D_x}\Big/\sqrt{1+\frac{D_z^2}{D_x^2}}\right)\right] \tag{7}$$

Introducing the concept of mean beam length (physically, it is the length of a corresponding one dimensional line-of-sight which yields the same transmissivity)

$$\tau_{d1-2} = \exp(-kL_m) \tag{8}$$

the mathematical expression of mean beam length is

$$L_m/D_x\left(\frac{D_y}{D_x},\frac{D_z}{D_x},kD_x\right) = -ln\left[\frac{(g_1+g_2+g_3)}{F_{d1-2}}\right]/(kD_x) \tag{9}$$

Note that the normalized mean beam length (L_m/D_x) is only a function of the optical thickness kD_x and the dimensionless geometric parameters, (D_y/D_x, D_z/D_x).

Figs. 2 show the effect of optical thickness and geometric effect on the mean beam lengths and the exchange factors for the 6 different geometric configurations. It is interesting to note that as the vertical dimension (D_z/D_x) increases, the MBL and the associated exchange factor approach a constant value. Physically, the upper portion of the area has decreasing contribution to the total radiative exchange as the vertical dimension increases. The MBL thus approaches an asymptotic constant value as D_z/D_x increases.

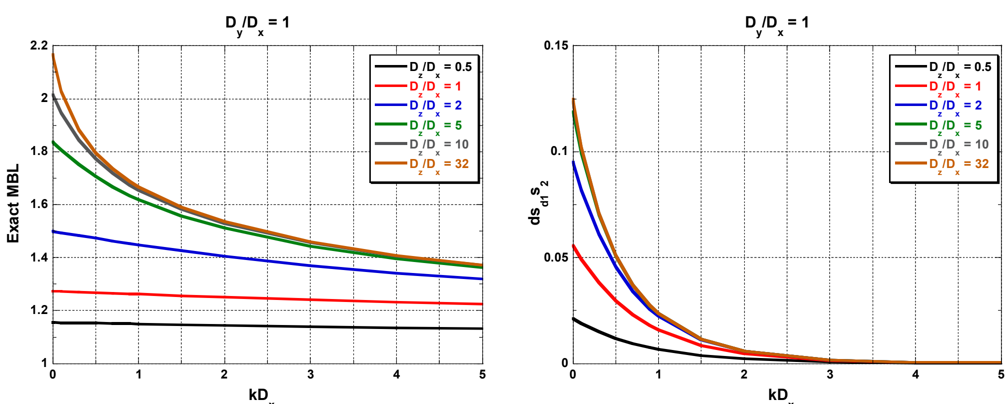

Fig. 2 a) Exact MBLs and b) exchange factors as a function of optical thickness for different D_z/D_x.

While the exact value of the MBL varies with optical thickness, numerical results show further that the exchange factor can be correlated accurately with a constant average MBL. Mathematically, this average MBL is defined by first introducing an accumulative error function for a general dimensionless variable η as follow

$$E(\eta) = \sum_{i=1}^{N}\left(\tau_{d1-2} - e^{-(kD_x)_i\eta}\right)^2 \quad (10)$$

where N is the total number of optical thicknesses being considered. The average MBL is defined to be the value of η at which the accumulative error function is a minimum. Mathematically, it is given by

$$\frac{dE}{d\eta}\left(\frac{L_{m,a}}{D_x}\right) = -\sum_{i=1}^{N} 2kD_x e^{-(kD_x)_i\left(\frac{L_{m,a}}{D_x}\right)}\left(\tau_{d1-2} - e^{-(kD_x)_i\left(\frac{L_{m,a}}{D_x}\right)}\right) = 0 \quad (11)$$

The average mean beam lengths, the exact exchange factors, and the approximated exchange factors for the 6 cases are listed in Table 1. It can be seen that a constant average MBL is an excellent approximation to the radiative heat transfer at different geometry and optical thickness.

Table 1 Relative difference in between exact and approximated exchange factor for different geometry and optical thickness with $D_y/D_x = 1.0$.

kD_x	D_z/D_x	Average MBL	Exchange Factor (Approximated)	Exchange Factor (Exact)	Relative Difference
0.1	1	1.2598	0.04914	0.04908	-0.00006
0.1	5	1.6411	0.10095	0.09930	-0.00165
0.1	10	1.6946	0.10419	0.10159	-0.00260
1.0	1	1.2598	0.01581	0.01578	-0.00003
1.0	5	1.6411	0.02305	0.02356	0.00051
1.0	10	1.6946	0.02267	0.02357	0.0009

Utilizing the concept of average MBL, the radiative heat transfer from a black differential area dA_1 with temperature T_w to a finite area A_2 as shown in Figs 1a and 1b, with an intervening combustion gas mixture, can now be written as

$$Q_{d1-2} = \sigma T_w^4 ds_1s_2 = dA_1F_{d1-2}\int_0^\infty e_{\lambda,b}(T_w)\tau_{d1-2,\lambda}d\lambda = \sigma T_w^4 dA_1F_{d1-2}\int_0^\infty e^{-k_\lambda L_{m,a}}d\lambda \quad (12)$$

Since the average MBL is constant for a given geometry, the spectral integral in Eq. (2) can be readily evaluated using RADNNET. For a given source temperature T_w, mixture temperature T_g, absorbing gas partial pressure P_g, mole fraction of CO_2 X_{CO_2}, and soot volume fraction f_v, the differential exchange factor can be written as

$$ds_1s_2 = dA_1F_{d1-2}\left[1 - \alpha_{d1-2}\left(T_g, T_w, P_gL_{m,a}, \chi_{co_2}, f_vL_{m,a}\right)\right] \quad (13)$$

where α is the total absorptivity evaluated based on RADNNET. It is important to note that for a rectangle with arbitrary dimension and location relative to the differential area dA_1, either in the parallel or perpendicular orientation, can be generated by a finite sum or differences of rectangle with the geometry as shown in Figs. 1a and 1b. The differential exchange factor between dA_1 and a rectangle of arbitrary dimension (either in the parallel or perpendicular orientation) can thus be written as a finite sum or differences with terms similar to Eq. (13). This is a great reduction in computational time and effort compared to direct numerical integration. For the radiative exchange between two finite rectangular areas A_1 and A_2 (either in the parallel or perpendicular orientation), the exchange factor can be generated by a single numerical integration. This is the basis of RADNNET-MBL.

3. RESULTS AND DISCUSSION

A verification case is provided to validate the accuracy and to examine the numerical efficiency of RADNNET-MBL for the evaluation of exchange factor with for two perpendicular surfaces in a 1m^3 cubical enclosure. The surfaces are back. The temperature of the surfaces and the mixture temperatures are uniform and maintained at 1000K. The total pressure of the gas mixture is kept at 1atm and it consists of water vapor and soot particulates.

Results with differential absorption gas pressure and soot volume fraction are generated and they are summarized in Table 2. It can be seen that the relative difference associated with the exact and the RADNNET-MBL generated surface-surface exchange factors is less than 1%. RADNNET-MBL, however, speeds up the evaluation dramatically.

Table 2 Summary of results for the verification cases.

Case	$T_g = T_w$ (K)	P_g	f_v	S_1S_2	CPU (s)	S_1S_2(MBL)	CPU (s)
1	1000	0	0	0.1999	1.562E-2	0.1999	1.562E-2
2	1000	0	5E-8	0.1901	3.125E-2	0.1908	1.562E-2
3	1000	30	0	0.1615	27.70	0.1630	7.813E-2
4	1000	30	5E-8	0.1539	6.547	0.1556	9.375E-2

4. CONCLUSIONS

RADNNET-MBL is presented. Based on numerical study, it is demonstrated that the new approach is computationally efficient and accurate for the evaluation of radiative heat transfer between arbitrary rectangular surfaces in a three-dimensional enclosure with a non-gray combustion medium. For four test cases, the relative difference associated with the surface-surface exchange factor is shown to be less than 1%. Using RADNNET-MBL, the evaluation of the exchange factor for some cases can be reduced by more than a factor of 100.

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