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Development of the concept of mean temperatures in the analysis of radiative heat transfer in an inhomogeneous non-isothermal non-gray medium

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ABSTRACT

Concepts of an emission mean temperature and a transmission mean temperature are introduced and shown to be effective and necessary in characterizing the radiative emission and transmission of an inhomogeneous non-isothermal non-gray medium. Based on a two zone model for a $CO_2/H_2O/N_2$ mixture, the mathematical behavior of these two mean temperatures and their dependence on the temperature and the partial pressure of the absorbing gases of the two zones are illustrated. Results show that the average temperature is not appropriate to be used in the prediction of radiative emission and transmission in practical engineering systems.

1. Introduction

The importance of radiation heat transfer in many practical engineering systems and the related problems such as combustion furnaces and enclosure fires is well known. Historically, the implementation of radiative heat transfer in practical engineering calculation is limited because of the mathematical complexity in accounting for the spectral and geometric effect of radiation. Over the past thirty years, a significant amount of research has been conducted and sophisticated solution methods have been developed. To account for the geometric effect, for example, there are the zonal method [1–3], the discrete ordinate method [4–6] and many others. To account for the spectral effect, there are the narrow band model [7], the *k*-distribution method [8] and the weighted gray gas method [9,10]. In recent years, methods are also developed to account for both the spectral and geometric effect simultaneously [11]. Indeed, with the increasing availability of fast computational power and large data storage capability, many practical engineering CFD (computational fluid dynamic) codes such as FLUENT [12] and FDS [13] have now integrated a radiation heat transfer solver using one of the solution methods. Highly sophisticated CFD simulations with impressive graphic results accounting for the effect of radiation heat transfer now appear to be available and these codes are being used more frequently and routinely in important engineering design and fire safety analysis.

Fundamentally, while an accurate solution method which can account for the spectral and geometric dependence of radiation heat transfer is important, the effect of the radiative and thermal properties of the participating medium is equally important and can have a significant impact on the accuracy of the results. All practical engineering systems are generally inhomogeneous and non-isothermal. Most, if not all, of the solution methods for radiation heat transfer, however, are developed under the assumption of an isothermal, homogenous medium. In the implementation of these methods in a practical engineering calculation, some "average" properties are generally used in the solver. Since the radiation effect is non-localized, the determination of the appropriate "average" properties used in the radiation solver is therefore a key to an accurate simulation of radiative heat transfer in practical engineering systems. Indeed, the presentation of results generated by CFD codes without a detailed explanation on how the "average" radiation properties are evaluated is meaningless since the accuracy of the results can be highly uncertain.

Over the past 30 years, the area of radiative heat transfer in inhomogeneous and non-isothermal medium has not received much attention. Indeed, the only reported research efforts on the analysis of radiative heat transfer in inhomogeneous non-isothermal gases appear to be those on the development of the Curtis-Godson approximation [14–18] in the 1960s. These efforts, however, were focused on obtaining appropriate "equivalent uniform" properties for individual absorption band. Since different "equivalent" properties are determined for different absorption bands, this

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Nomenclature

a_λ	absorption coefficient
d_1	length scale of zone 1 (m), Fig. 1
d_2	length scale of zone 2 (m), Fig. 1
d_m	length scale of the equivalent homogeneous isothermal zone (m), Fig. 1
$e_{\lambda b}$	Planck function
P_{H_2O}	partial pressure of H_2O (kPa)
P_{CO_2}	partial pressure of CO_2 (kPa)
$Q_{ih,e}$	radiative heat flux emitted from the inhomogeneous mixture from the surface of zone 1
$Q_{h,e}$	radiative heat flux emitted from the equivalent homogeneous mixture
T	temperature (K)
$T_{m,em}$	emission mean temperature (K)
$T_{m,tr}$	transmission mean temperature (K)
τ_{ih}	transmissivity of the inhomogeneous mixture
τ_h	transmissivity of the equivalent homogeneous mixture

Subscripts

1, 2	label for two zones of the inhomogeneous mixture
m	label for the equivalent homogeneous zone
λ	wavelength

concept is not feasible for the determination of total heat transfer in a practical engineering system.

The objective of this work is to introduce the concept of two mean temperatures and the associated mean partial pressures for an inhomogeneous non-isothermal medium so that existing radiation solvers can be used accurately and efficiently in practical engineering calculation. Similar to the concept used by the Curtis-Godson approximation, the "equivalent" mean temperature and partial pressure are defined as the properties such that the total transmission through this "equivalent isothermal gas" and its total emission are identical to those of a given path consisting of an inhomogeneous non-isothermal gas. In this work, this concept is illustrated by a two-zone $CO_2/H_2O/N_2$ mixture. The numerical data are generated using an existing well established narrow-band radiation solver RADCAL [7]. The two mean temperatures are tabulated numerically for different gas properties and compositions to illustrate their dependence on the mixture properties. Based on these data, the error of the common approach which uses a volumetric average temperature in predicting the radiation effect is assessed.

2. Mathematical formulation

A two-zone non-isothermal inhomogeneous medium as shown in Fig. 1 will be used as a basis for the development of the "radiation mean" temperature. The medium is assumed to be a $CO_2/H_2O/N_2$ mixture at atmospheric pressure. The physical dimension for each zone will be assumed to be 1 m. For a given set of conditions (T_1, $P_{CO_2,1}d_1$, $P_{H_2O,1}d_1$, T_2, $P_{CO_2,2}d_2$, $P_{H_2O,2}d_2$) of the two-zone mixture, a mean temperature will be determined by finding the temperature of an equivalent isothermal, homogeneous medium as shown in Fig. 1, with conditions T_m, $P_{CO_2,m}d_m$, $P_{H_2O,m}d_m$, which yield the same radiative heat transfer result as the original mixture. For simplicity, the current work will consider only mixtures maintained at atmospheric pressure. Extension to higher pressure is straight forward and can be considered in future works.

The "equivalent" mixture is assumed to be at atmospheric pressure. The total mass of the absorbing gases, CO_2 and H_2O, will be assumed to be equal to those of the original two zones. Based on the mean temperature T_m and treating the mixture as an ideal gas, the partial pressure and the physical dimension of the equivalent isothermal and homogeneous medium is given by

$$d_m = T_m\left(\frac{d_1}{T_1}+\frac{d_2}{T_2}\right) \tag{1}$$

$$P_{H_2O,m}d_m = T_m\left(\frac{P_{H_2O,1}d_1}{T_1}+\frac{P_{H_2O,2}d_2}{T_2}\right) \tag{2}$$

$$P_{CO_2,m}d_m = T_m\left(\frac{P_{CO_2,1}d_1}{T_1}+\frac{P_{CO_2,2}d_2}{T_2}\right) \tag{3}$$

Since a different integration is involved in the evaluation of the emission and absorption of the mixture, a different mean temperature will be introduced for the two processes.

For the evaluation of emission from the two zone mixture radiating from the surface of the first zone as shown in Fig. 1, the total emission is given by

$$\begin{aligned} Q_{ih,e} = {} & \int_0^\infty e_{\lambda b}(T_1)\left\{1-\exp\left[-a_\lambda(T_1,P_{H_2O,1},P_{CO_2,1})d_1\right]\right\}d\lambda \\ & + \int_0^\infty e_{\lambda b}(T_2)\exp\left[-a_\lambda(T_1,P_{H_2O,1},P_{CO_2,1})d_1\right] \\ & \times\left\{1-\exp\left[-a_\lambda(T_2,P_{H_2O,2},P_{CO_2,2})d_2\right]\right\}d\lambda \end{aligned} \tag{4}$$

The emission for the equivalent one-zone homogenous medium is

$$Q_{h,e} = \int_0^\infty e_{\lambda b}(T_{m,em})\left\{1-\exp\left[-a_\lambda(T_{m,em},P_{H_2O,m},P_{CO_2,m})d_m\right]\right\}d\lambda \tag{5}$$

Eqs. (4) and (5) can be evaluated with any spectral integration method (e.g. line by line, narrow band model, etc.). By setting $Q_{ih,e} = Q_{h,e}$, a mean emission temperature, $T_{m,em}$, can be readily tabulated as a function of the mixture properties. Note that the concept of a mixture emissivity is meaningless for a non-isothermal mixture since it is a function of the assumed mixture temperature. The emission mean temperature is therefore determined based on the equivalence of the total radiative emission, not on the equivalence of the mixture emissivity.

For the evaluation of transmission through the two zone mixture, the following expression for the transmissivity is evaluated

$$\begin{aligned} \tau_{ih} = {} & \int_0^\infty e_{\lambda b}(T_s) \\ & \times\exp\left[-a_\lambda(T_1,P_{H_2O,1},P_{CO_2,1})d_1 - a_\lambda(T_2,P_{H_2O,2},P_{CO_2,2})d_2\right]d\lambda \end{aligned} \tag{6}$$

For the one-zone homogenous medium, the transmissivity is

$$\tau_h = \int_0^\infty e_{\lambda b}(T_S)\exp\left[-a_\lambda(T_{m,tr},P_{H_2O,m},P_{CO_2,m})d_m\right]d\lambda \tag{7}$$

By setting $\tau_{ih} = \tau_h$, a mean transmission temperature, $T_{m,tr}$, can be tabulated. Note that $T_{m,tr}$ is generally a function of both the mixture properties and the source temperature.

The two different mean temperatures can be readily evaluated using any spectral radiation solver. The current work uses the narrow band model, RADCAL [7], which has been shown to be sufficiently accurate for application in combustion media at atmospheric conditions.

3. Results and discussion

3.1. The emission mean temperature

The emission mean temperature for a H_2O/N_2 mixture with different partial pressures and temperature at the two zones are tabulated and shown in Fig. 2(a–d). The H_2O partial pressure in zone 1 is kept constant at 1.0 kPa-m to show the effect of increasing H_2O partial pressure in zone 2. The average temperature, $T_{avg} = (T_1 + T_2)/2$, is shown in the same figure for comparison. It is apparent that except for the case of a homogeneous medium ($P_{H_2O,1}d_1 = P_{H_2O,2}d_2 = 1.0$ kPa-m), the average temperature is not a good approximation of the emission mean temperature and it therefore does not yield an accurate prediction of the emission. As the optical thickness of zone 2 increases, the emission mean temperature approaches the temperature of zone 2, as the emission is dominated by the optically thick region. To demonstrate the error of using the average temperature in predicting the heat transfer, the emission based on the average temperature (with the "average" partial pressure computed using expressions similar to Eqs. (2) and (3)) are compared with the exact results in Fig. 3(a–d). It can be readily observed that the error of using the average temperature to calculate the total emission can be quite substantial, up to an order of magnitude or more in some cases.

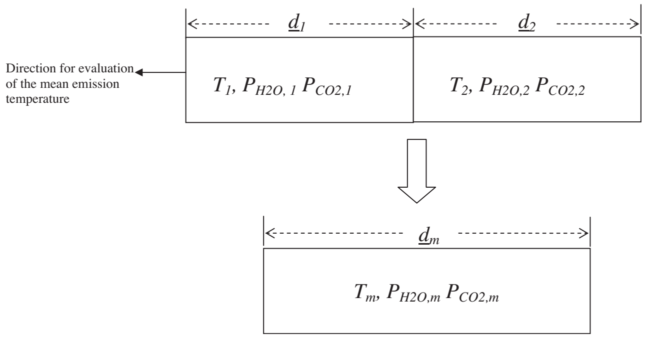


Fig. 1. Geometry and properties of the two-zone system and the equivalent one-zone system used in the definition of the mean temperature.

To investigate the behavior of a homogeneous non-isothermal medium, the emission mean temperature for a homogeneous H_2O/N_2 mixture with different partial pressures and temperature is shown in Fig. 4(a–d). It can be readily observed that in the optically thin limit, the average temperature is a good approximation for the mean emission temperature. When the optical thickness increases, the mean emission temperature deviates significantly from the average temperature. The corresponding difference in the emission is shown in Fig. 5(a–d). In the highly non-isothermal cases ($T_1 = 300$ K, $T_2 = 2000$ K or $T_1 = 2000$ K, $T_2 = 300$ K), the difference in the radiative emission is quite significant.

To illustrate the effect of different species concentration, the emission mean temperature for an inhomogeneous $H_2O/CO_2/N_2$ mixture with different partial pressures and temperature is shown in Fig. 6(a–d). The first zone is taken to be a H_2O/N_2 mixture with a constant partial pressure of 1.0 kPa-m. The second zone is taken to be a CO_2/N_2 mixture with different partial pressures. It is interesting to note that, for this type of mixture, the average temperature is not a good approximation of the emission mean temperature at any condition. As expected, in the limit of a large partial pressure for CO_2 in the zone 2, the emission mean temperature approaches the zone 2 temperature, T_2. The influence of the zone 1 temperature on the emission mean temperature is stronger than that for the case of a H_2O/N_2 mixture (Fig. 4(a–d)), due to the decreasing

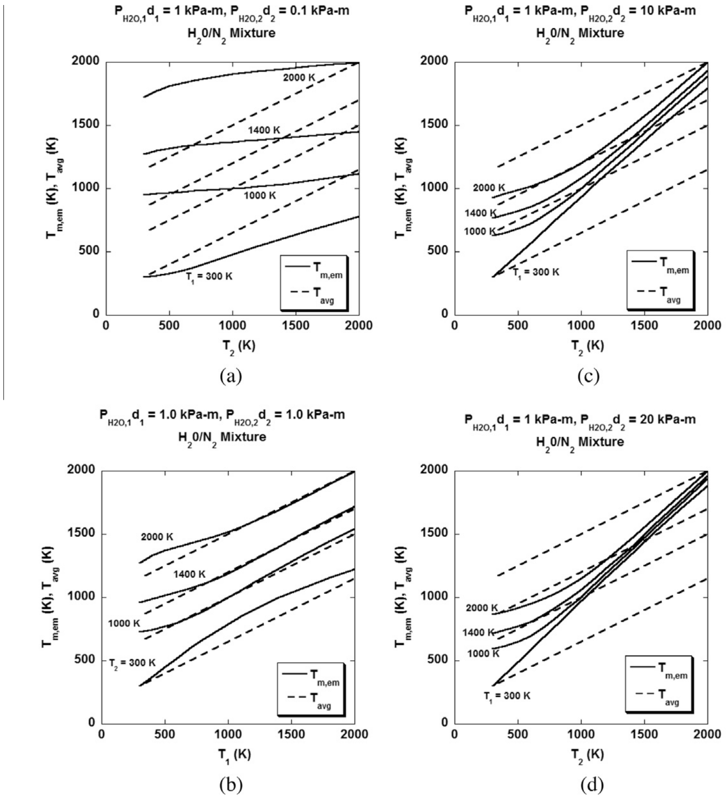
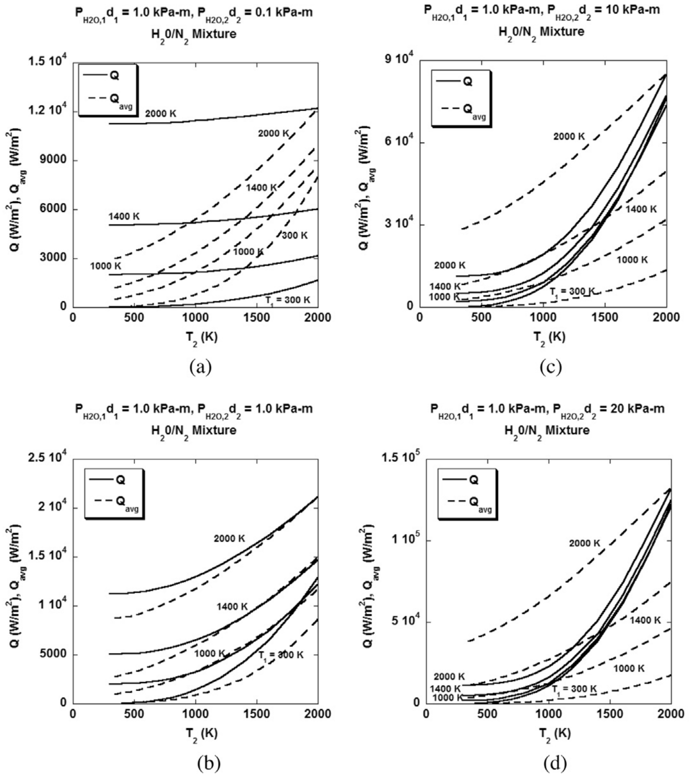


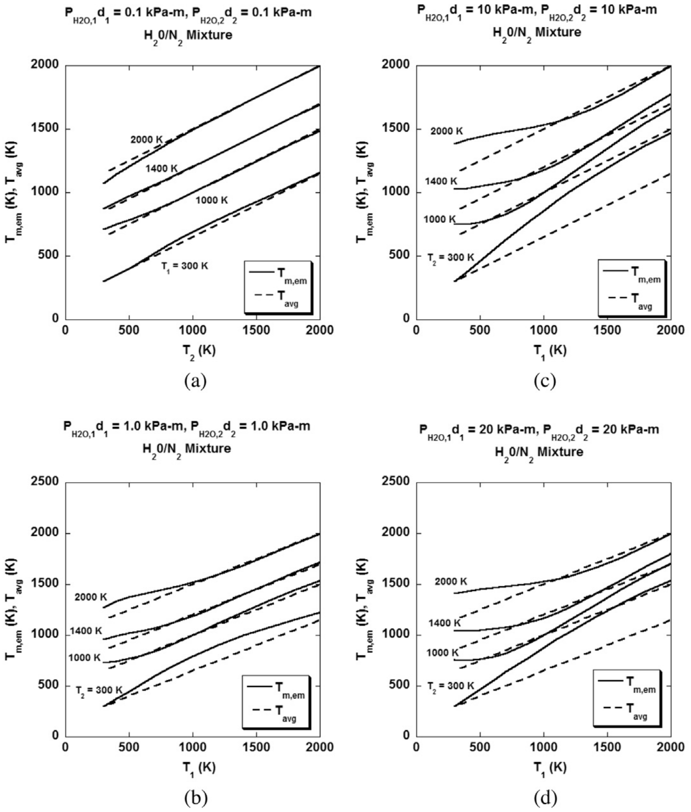
Fig. 2. The emission mean temperature for an H_2O/N_2 mixture with the partial pressure in zone 1 kept constant at $P_{H_2O,1}d_1 = 1.0$ kPa-m. The average temperature (for different T_1) is plotted as broken lines in each figure for comparison.



(a) (c)

(b) (d)

Fig. 3. The radiative heat flux emitted from an H_2O/N_2 mixture with the partial pressure in zone 1 kept constant at $P_{H_2O,1}d_1$ = 1.0 kPa-m. The heat flux calculated based on the average temperature (for different T_1) is plotted as broken lines in each figure for comparison.



(a) (c)

(b) (d)

Fig. 4. The emission mean temperature for a homogeneous, non-isothermal H_2O/N_2 mixture with different partial pressures. The average temperature is plotted as the broken line in each figure for comparison.

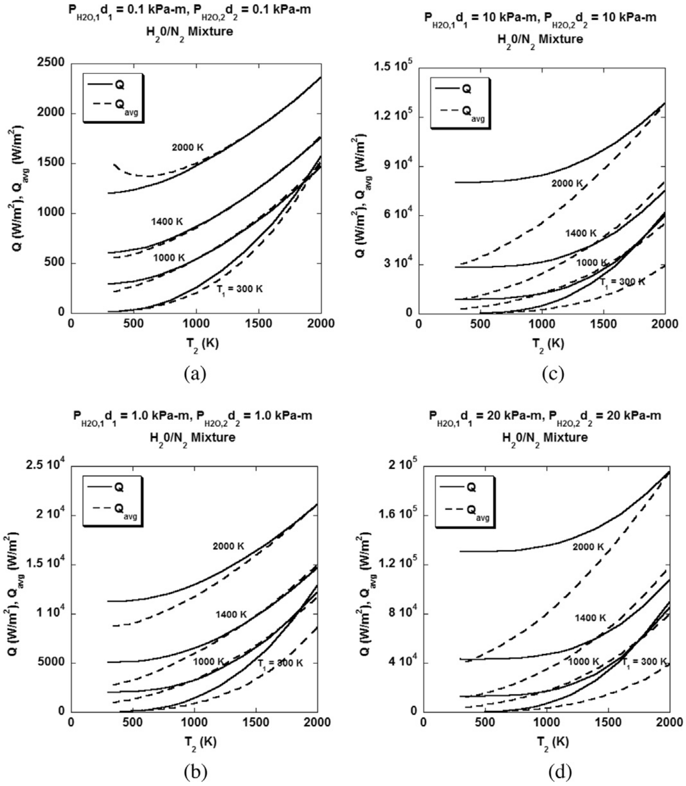


Fig. 5. The radiative heat flux emitted from a homogeneous and non-isothermal H_2O/N_2 mixture with different partial pressure. The heat flux calculated based on the average temperature (for different T_1) is plotted as broken lines in each figure for comparison.

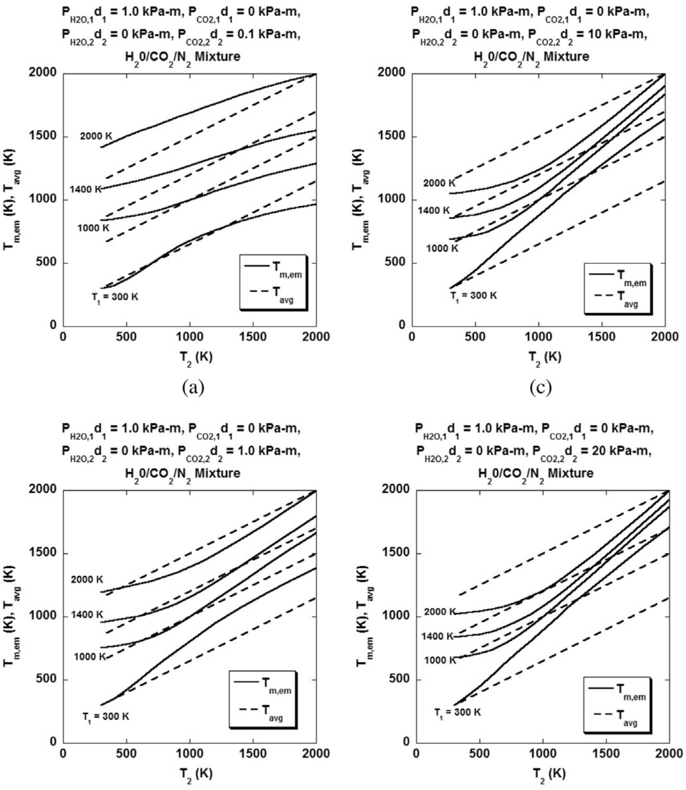


Fig. 6. The emission mean temperature for an $H_2O/CO_2/N_2$ mixture with zone 1 being a H_2O/N_2 mixture and zone 2 a CO_2/N_2 mixture. The partial pressure in zone 1 kept constant at $P_{H_2O,1}d_1$ = 1.0 kPa-m. The average temperature (for different T_1) is plotted as broken lines in each figure for comparison.

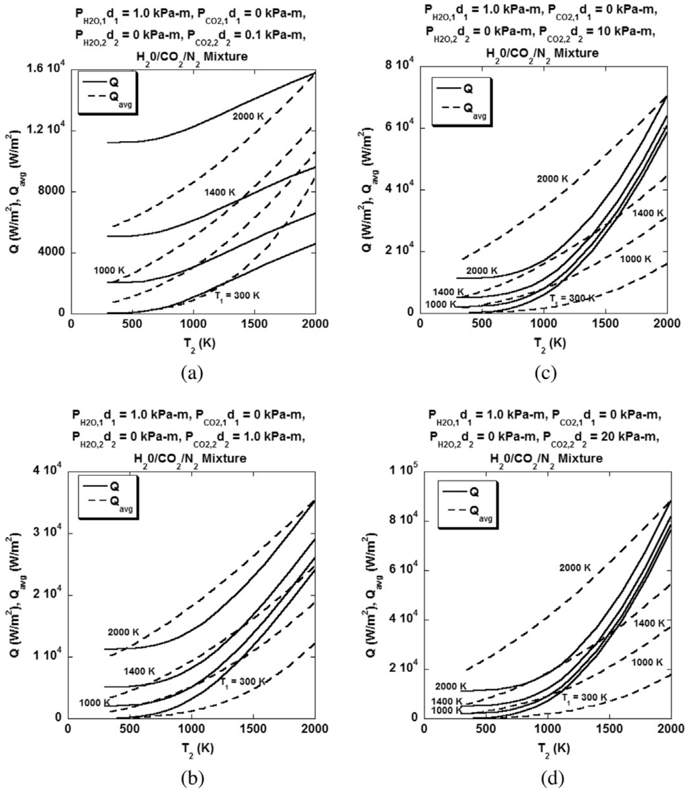


Fig. 7. The radiative heat flux emitted from a $H_2O/CO_2/N_2$ mixture with zone 1 being a H_2O/N_2 mixture and zone 2 a CO_2/N_2 mixture. The partial pressure in zone 1 kept constant at $P_{H_2O,1}d_1$ = 1.0 kPa-m. The heat flux calculated based on the average temperature (for different T_1) is plotted as broken lines in each figure for comparison.

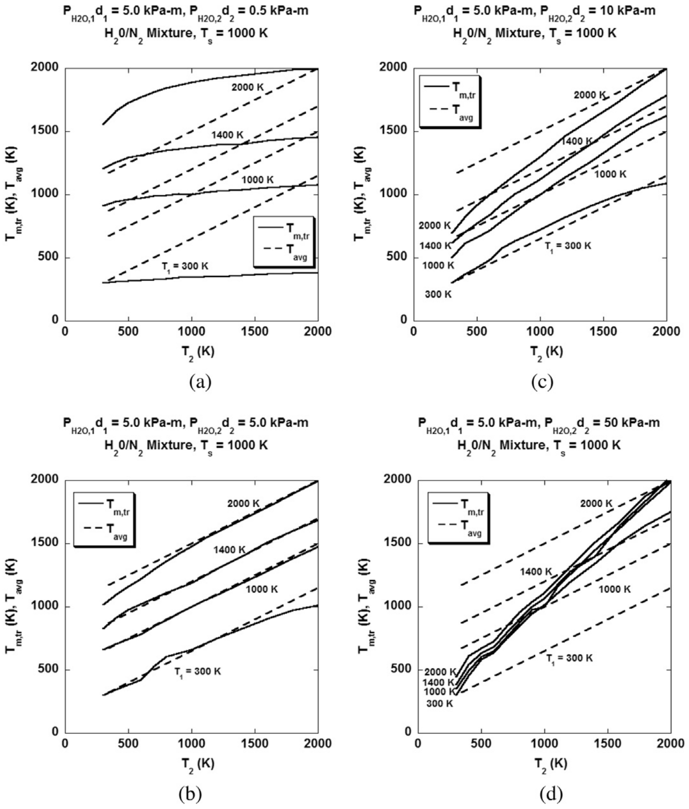
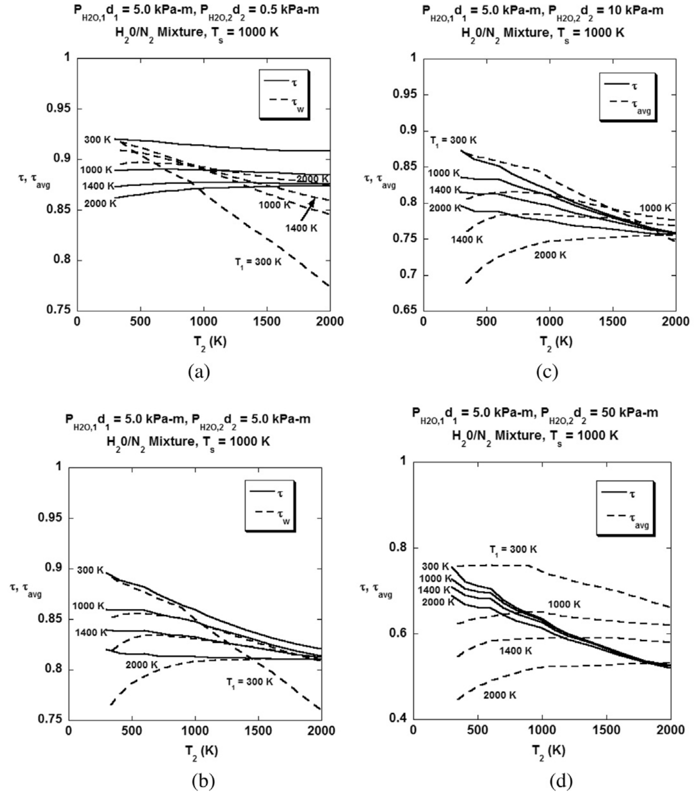


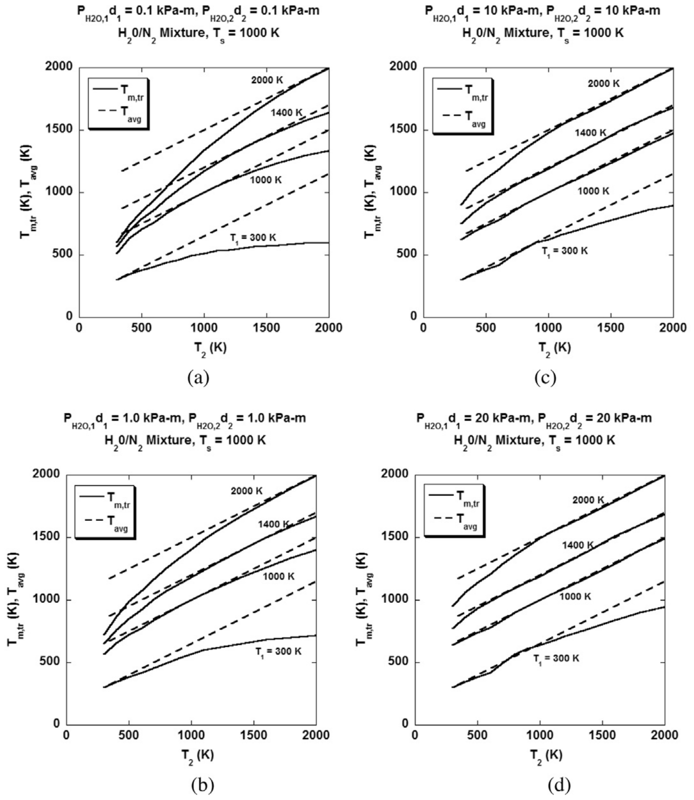
Fig. 8. The transmission mean temperature for an H_2O/N_2 mixture with the partial pressure in zone 1 kept constant at $P_{H_2O,1}d_1$ = 5.0 kPa-m. The average temperature (for different T_1) is plotted as broken lines in each figure for comparison.



(a) (c)

(b) (d)

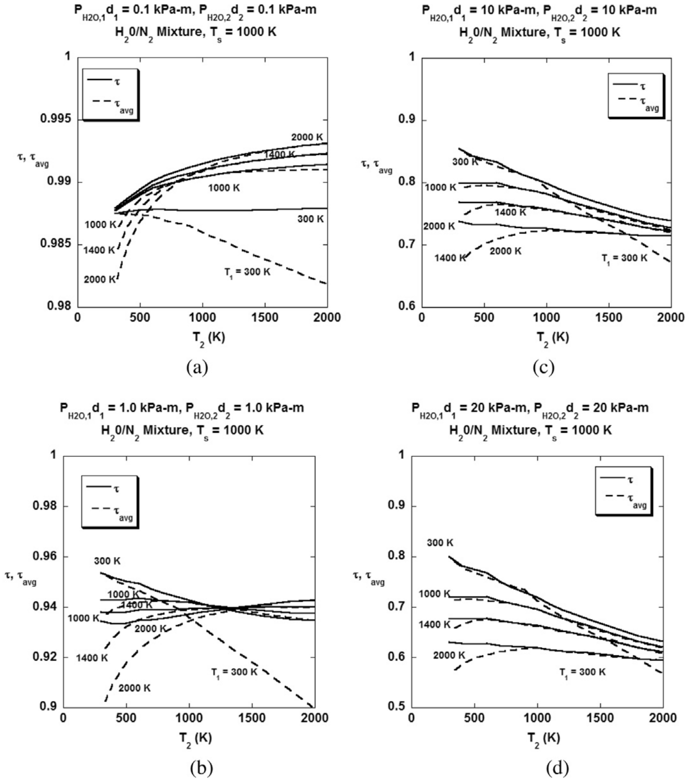
Fig. 9. The transmissivity for an H_2O/N_2 mixture with the partial pressure in zone 1 kept constant at $P_{H_2O,1}d_1$ = 5.0 kPa-m. The transmissivity calculated based on the average temperature (for different T_1) is plotted as broken lines in each figure for comparison.



(a) (c)

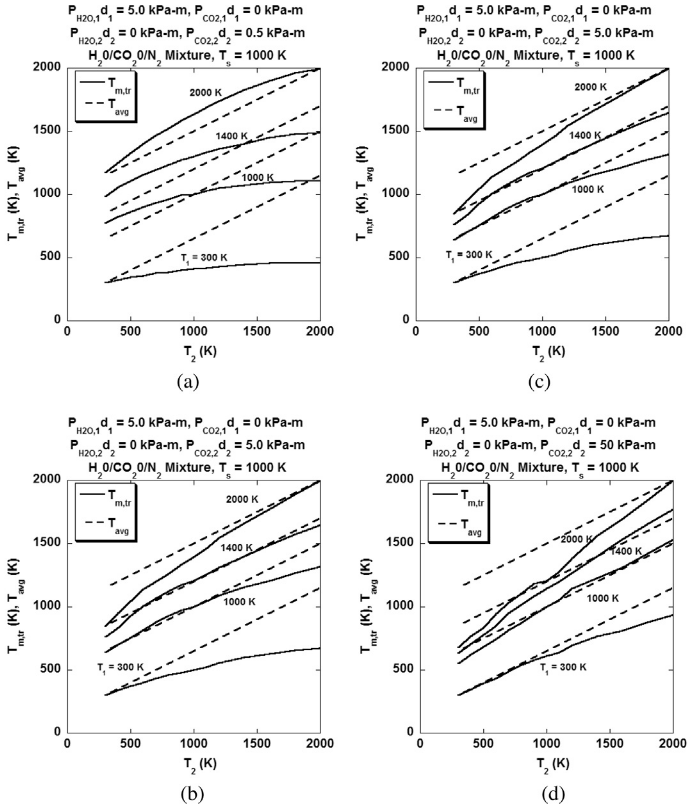
(b) (d)

Fig. 10. The transmission mean temperature for a homogeneous and non-isothermal H_2O/N_2 mixture with different partial pressure. The average temperature (for different T_1) is plotted as broken lines in each figure for comparison.



(a) (c)
(b) (d)

Fig. 11. The transmissivity for a homogeneous and non-isothermal H_2O/N_2 mixture at different partial pressure. The transmissivity calculated based on the average temperature (for different T_1) is plotted as broken lines in each figure for comparison.



(a) (c)
(b) (d)

Fig. 12. The transmission mean temperature for a $H_2O/CO_2/N_2$ mixture with zone 1 being a H_2O/N_2 mixture and zone 2 a CO_2/N_2 mixture. The partial pressure in zone1 kept constant at $P_{H_2O,1}d_1$ = 5.0 kPa-m. The average temperature (for different T_1) is plotted as broken lines in each figure for comparison.

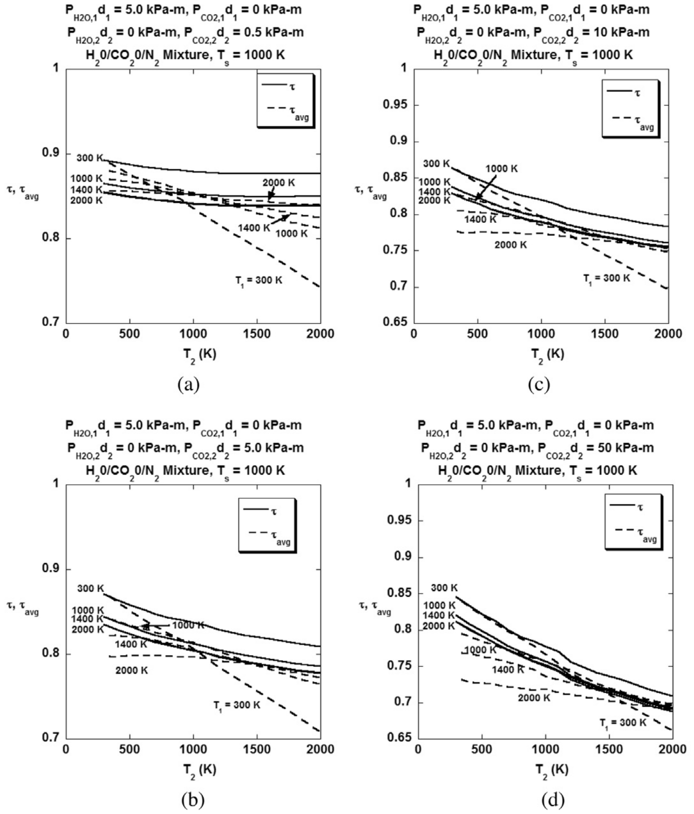


Fig. 13. The transmissivity for a $H_2O/CO_2/N_2$ mixture with zone 1 being a H_2O/N_2 mixture and zone 2 a CO_2/N_2 mixture. The partial pressure in zone 1 kept constant at $P_{H_2O,1}d_1$ = 5.0 kPa-m. The transmissivity calculated based on the average temperature (for different T_1) is plotted as broken lines in each figure for comparison.

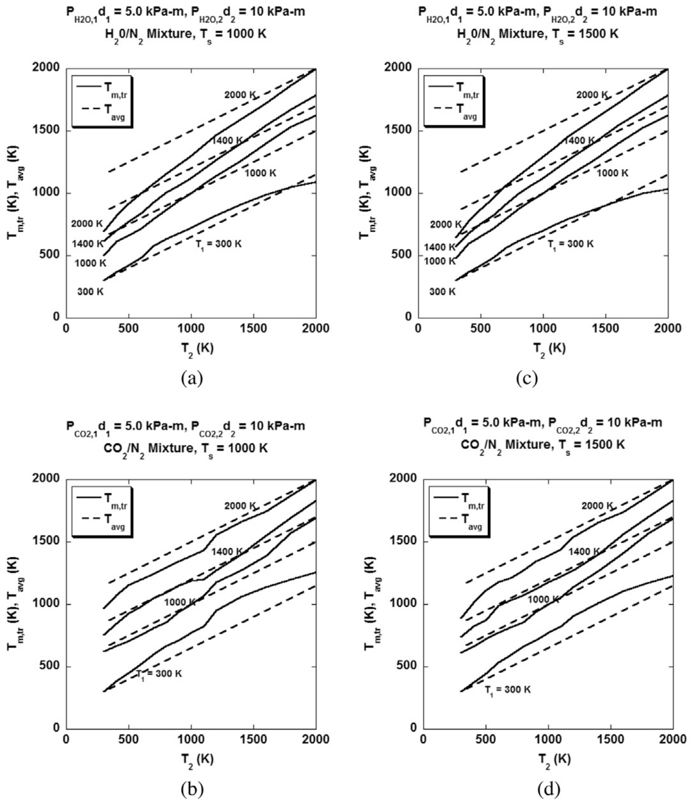


Fig. 14. The effective of the source temperature on the transmission mean temperature for an inhomogeneous, non-isothermal H_2O/N_2 mixture (a and c) and an inhomogeneous non-isothermal CO_2/N_2 mixture (b and d). The average temperature (for different T_1) is plotted as broken lines in each figure for comparison.

emissive power of CO_2. The corresponding radiative emission for the mixture is shown in Fig. 7(a–d).

In summary, the emission mean temperature depends strongly on the mixture properties and temperature. In a simple two zone system and in the limit of one zone having a much larger optical thickness than the other, the emissive mean temperature approaches the temperature of the optically thick zone. Except for an optically thin homogeneous mixture, the average temperature is not a good approximation of the emissive mean temperature. The use of an average temperature generally leads to significant error in the prediction of radiative emission from an inhomogeneous, non-isothermal mixture.

3.2. The transmission mean temperature

The transmission mean temperature for a H_2O/N_2 mixture at difference mixture conditions with a source temperature T_s of 1000 K is shown in Fig. 8(a–d). To show a higher effect of radiative absorption, the H_2O partial pressure in zone 1 is kept constant at 5.0 kPa-m. When the partial pressure of the absorbing gas in one region is much higher than the other, the transmission mean temperature takes on the value of the temperature of the region of the higher pressure (T_1 for data in Fig. 8a and T_2 for data in Fig. 8d). For cases with the absorbing partial pressure of the two regions being comparable (Fig. 8b and c), the transmission mean temperature deviates from both zone temperature. The error of using average temperature to calculate the transmissivity is substantial, as shown in results presented in Fig. 9(a–d).

For transmission mean temperature for a homogenous H_2O/N_2 mixture at difference partial pressure with a source temperature T_s of 1000 K is shown in Fig. 10(a–d). Unlike the emission mean temperature, the transmission mean temperature approaches the average temperature in the optically thick limit. But in the case of large temperature differences ($T_1 = 300$ K, $T_2 = 2000$ K and $T_1 = 2000$ K, $T_2 = 300$ K), the difference between the transmission mean temperature and the average temperature is quite significant. The difference of the corresponding transmissivities, as shown in Fig. 11(a–d), is also quite large.

The transmission mean temperature for an inhomogeneous $H_2O/CO_2/N_2$ mixture with different partial pressures and temperature is shown in Fig. 12(a–d) and the corresponding transmissivity is shown in Fig. 13(a–d). It is interesting to note that the results do not show any correlation with the two zone temperature or the average temperature. The influence of the first zone temperature, T_1, on the transmissivity appears to become less significant as the optical thickness of the second zone becomes large.

The effect of the source temperature on the transmission mean temperature is shown in Fig. 14(a–d). It is interesting to note that the source temperature has only a minimal effect on the transmission mean temperature, for both CO_2/N_2 and H_2O/N_2 mixtures. In general, numerical data show that the transmission mean temperature is largely independent of source temperature, provided that the source temperature is in the range such that the emission includes the relevant absorption bands of CO_2 and H_2O. While the source temperature can have some effect on the transmission mean temperature when the emission is outside of the relevant absorption bands, its effect of the calculation of transmissivity is unimportant because the mixture is optically transparent and the transmissivity is close to unity.

In summary, the transmission mean temperature for a gas mixture is generally different from the emission mean temperature. When the optical thickness of one zone is significantly higher than the second zone, the transmission mean temperature approaches the temperature of the optically thick zone. The average temperature is a good approximation of the transmission mean temperature only for a homogenous optically thick medium with small variation of temperature between the two zones. The source temperature does not have a significant effect on the transmission mean temperature.

4. Conclusion

The radiative heat transfer in an inhomogeneous, non-isothermal medium is shown to be effectively characterized by the concept of an emission mean temperature and a transmission mean temperature. Numerical data for the two mean temperatures are tabulated for a two-zone inhomogeneous $CO_2/H_2O/N_2$ mixture using RADCAL and their dependence on the mixture properties is illustrated. In general, the average zone temperatures is shown to be ineffective in characterizing the radiative heat transfer and can lead to significant error in the prediction of emission and transmissivity of an inhomogeneous, non-isothermal medium.

For practical application, these two mean temperatures should be tabulated and made available for implementation in various CFD codes with radiation solver. The access to these data can be done efficiently and accurately with the development of a neural network (trained by the numerical data) similar to the approach used in reference [11]. This work is currently underway and results will be presented in future publications.

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